

Gravitational Fields

1. Explain how Newton's law of gravitation is applied between two non-spherical asteroids.

Force is proportional to the product of the mass of each asteroid. and the force is inversely proportional to the distance squared between the centres of mass of the asteroids.

[1]

2. Write an expression for the gravitational potential V_g at the surface of a planet of mass M and radius r .

$$V_g = -G \frac{M}{r}$$

[1]

3. Fig. 22 shows the elliptical orbit of a planet around the Sun.

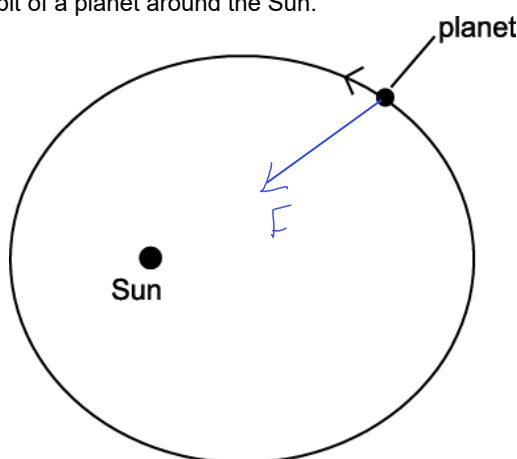


Fig. 22

Draw the gravitational force acting on the planet at the position shown in Fig. 22.

[1]

4. Kepler-90 is a star with several planets orbiting it.

The two outermost planets are Kepler-90g and Kepler-90h.

Kepler-90g has an orbital period of 210 days and is 0.71 AU from the centre of Kepler-90.

Kepler-90h is 1.01 AU from the centre of Kepler-90.

$$T_g = 210 \text{ days} \quad T_h = ?$$

$$r_g = 0.71 \text{ AU} \quad r_h = 1.01 \text{ AU}$$

Kepler's third law of planetary motion can be applied to the planets of Kepler-90.

What is the orbital period of Kepler-90h?

- A 50 days
B 299 days
C 356 days
D 4350 days

Your answer

$$T^2 = k r^3 \rightarrow k = \frac{T^2}{r^3} \Rightarrow \frac{T_g^2}{r_g^3} = \frac{T_h^2}{r_h^3}$$

$$\rightarrow T_h = \sqrt{\frac{T_g^2 r_h^3}{r_g^3}} = \sqrt{\frac{210^2 \times 1.01^3}{0.71^3}} = 356.3 \rightarrow C$$

Kepler's 3rd Law: $T^2 \propto r^3$
 $\rightarrow T^2 = k r^3$

We know $T^2 = \frac{4\pi^2}{Gm} r^3$

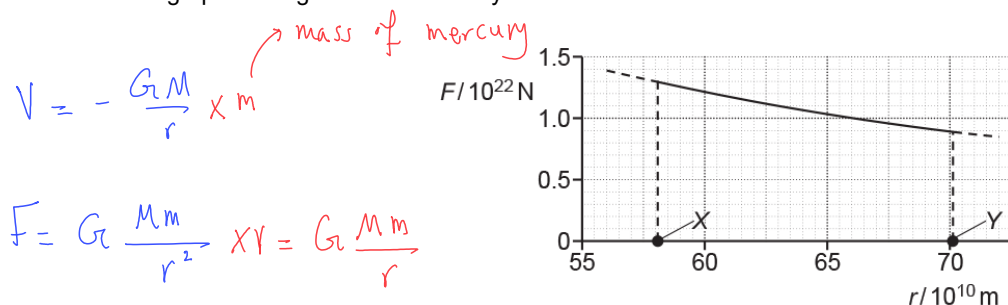
so $k = \frac{4\pi^2}{Gm}$ but we don't need to use this here.

[1]

5.4 Gravitational Fields

5. The planet Mercury has a highly elliptical orbit around the Sun.

The gravitational force F acting on Mercury due to the Sun varies with its distance r from the centre of the Sun. The graph of F against r for Mercury in its orbit is shown below.



Mercury is closest to the Sun when $r = X$ and furthest when $r = Y$.

What does the **area** under the graph between the distances X and Y represent?

- A The centripetal force acting on Mercury.
- B The change in the gravitational potential energy of Mercury.
- C The impulse of the force acting on Mercury.
- D The kinetic energy of Mercury.

$$V = \frac{2\pi r}{T}$$

Your answer

☐

$$\frac{1}{2} m v^2 \neq F \times r = \frac{GMm}{r^2} \times r$$

$$\frac{1}{2} m \left(\frac{2\pi r}{T} \right)^2 \neq \frac{GMm}{r^2}$$

[1]

6. Earth has a mass of $6.0 \times 10^{24} \text{ kg}$ and a radius of 6400 km.

A satellite of mass 320 kg is lifted from the Earth's surface to an orbit 1200 km above its surface.

What is the change in the gravitational potential energy of the satellite?

- A $9.1 \times 10^2 \text{ J}$
- B $9.9 \times 10^6 \text{ J}$
- C $3.2 \times 10^9 \text{ J}$
- D $3.8 \times 10^9 \text{ J}$

$$V = -\frac{GM}{r} \quad G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2} \quad M = 6 \times 10^{24} \text{ kg}$$

at surface of the Earth: $V_1 = -\frac{GM}{6400 \times 10^3} = -62531250$

at the orbit: $V_2 = -\frac{GM}{(6400+1200) \times 10^3} = -52657894.74$

Your answer

☐

[1]

$$\Delta V = V_2 - V_1 = 9.87 \times 10^6 \approx 9.9 \times 10^6 \text{ J this is per unit mass}$$

7. The Earth is surrounded by a gravitational field.

$$\rightarrow 9.9 \times 10^6 \times 320 = 3.15 \times 10^9 \approx 3.2 \times 10^9 \text{ J}$$

Which of the following statements is/are correct about the gravitational field lines near the surface of the Earth.

- 1 They are parallel.
- 2 They show the direction of the force on a small mass.
- 3 They are equally spaced.

- A Only 1
- B Only 1 and 2
- C Only 2 and 3
- D 1, 2 and 3

Your answer

☐

Near to the surface of the Earth, the gravitational field is approximately uniform.

This means that they are parallel and equally spaced.

Gravitational field lines in general show the direction of the force on a small mass. This makes all 3

statements are correct, giving the answer D.

[1]

5.4 Gravitational Fields

8. A satellite is in a circular orbit around the Earth. The vertical **height** of the satellite above the surface of the Earth is 3200 km. The radius of the Earth is 6400 km.

What is the ratio

weight of satellite in orbit
weight of satellite on the Earth's surface?

at height $\leftarrow g_2 = -\frac{GM}{r^2} = -\frac{GM}{(3200+6400)^2}$

at surface of earth $\leftarrow g_1 = -\frac{GM}{(6400)^2}$

$$\frac{g_2}{g_1} = \frac{-\frac{GM}{(3200+6400)^2}}{-\frac{GM}{(6400)^2}} = \frac{(6400)^2}{(3200+6400)^2} = \frac{1}{(1.5)^2} = \frac{1}{2.25} = 0.44 \rightarrow B$$

- A 0.25
B 0.44
C 0.50
D 0.67

Your answer

... $\rightarrow \frac{1}{(3200+6400)^2} \times \frac{(6400)^2}{1} = 0.44 \rightarrow B$ [1]

9. A satellite of mass 3000 kg moves from a parking orbit of radius 6800 km to a geostationary orbit of radius 42 000 km. The mass of the Earth is 6.0×10^{24} kg.

What is the magnitude of the change in gravitational potential?

- A. 8.4 J kg^{-1}
B. $2.5 \times 10^4 \text{ J kg}^{-1}$
C. $4.9 \times 10^7 \text{ J kg}^{-1}$
D. $1.5 \times 10^{11} \text{ J kg}^{-1}$

Your answer

[1]

10. This question is about a space probe which is in orbit around the Sun.

Define **gravitational potential energy** of an object at a point in a gravitational field.

[1]

11. Planets **X** and **Y** each have a single moon.

Planet **X** has twice the mass of planet **Y**. The orbital radius of the moon around planet **X** is three times the orbital radius of the moon around planet **Y**.

The gravitational potential of the planet **X** is V_X at the position of its moon. The gravitational potential of the planet **Y** is V_Y at the position of its moon.

What is the value of the ratio $\frac{V_X}{V_Y}$?

- A 0.22
B 0.67
C 1.50
D 6.00

Your answer

[1]

5.4 Gravitational Fields

12. The gravitational force between two point-mass objects **X** and **Y** is F_1 .

The mass of **X** increases and the distance between **X** and **Y** is halved.

Which statement about the new gravitational force F_2 between these two objects is correct?

- A $0 < F_2 < 0.25F_1$
- B $F_2 > 4F_1$
- C $F_2 = F_1$
- D $2F_1 < F_2 < 4F_1$

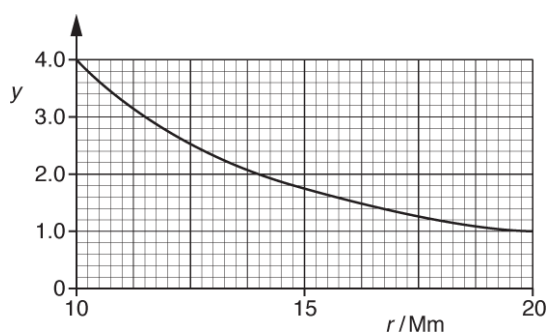
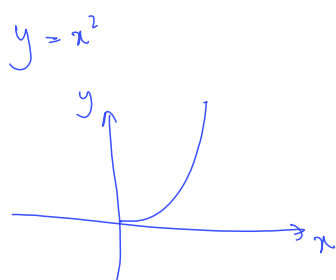
$$F = G \frac{m_x m_y}{(\frac{1}{2}r)^2} \rightarrow B$$

Your answer

☐

[1]

13. A graph of y against distance r from the centre of a planet is shown below.



$y \propto \frac{1}{r^2}$

either $F = G \frac{m_1 m_2}{r^2}$

or $g = G \frac{M}{r^2} \rightarrow C$

The graph shows that y is inversely proportional to r^2 .

Which quantity is best represented on the y -axis of the graph?

- A Period of a satellite orbiting the planet.
- B Gravitational potential of the planet.
- C Gravitational field strength of the planet.
- D Kinetic energy of a satellite orbiting the planet.

Your answer

☐

[1]

14. Scientists are planning to launch a rocket from the surface of the Earth into an orbit at a distance of 18000 km above the centre of the Earth. The radius of the Earth is 6400 km and it has mass 6.0×10^{24} kg.

What is the minimum work done to move the 150 kg mass of the rocket into this orbit?

- A -13×10^5 J
- B -6.0×10^5 J
- C $+6.0 \times 10^5$ J
- D $+13 \times 10^5$ J

Your answer

☐

[1]

5.4 Gravitational Fields

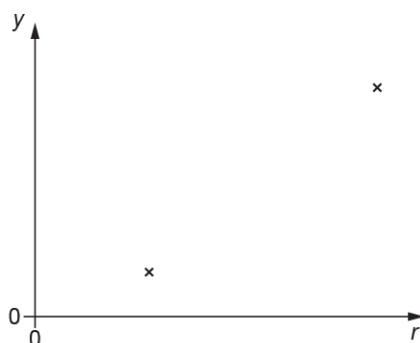
15. Which of the following is a correct unit for gravitational field strength?

- A J kg^{-1}
- B N kg^{-1}
- C $\text{N m}^2 \text{kg}^{-2}$
- D kg m s^{-1}

Your answer

[1]

16. A student has collected some data on the Solar System.
The student plots a graph, but only two data points are shown below.



The distance from the centre of the Sun is r .

Which quantity y is represented on the vertical axis?

- A Speed of a planet.
- B Period of a planet.
- C Gravitational potential of the Sun.
- D Gravitational field strength of the Sun.

Your answer

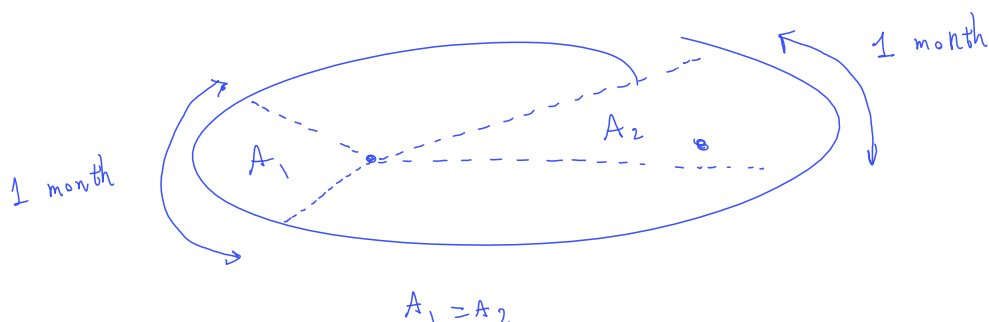
[1]

5.4 Gravitational Fields

17. Observations of the planets led to Kepler's three empirical laws:

1. The orbit of a planet is an ellipse with the Sun at one focus.
2. A line joining a planet and the Sun sweeps out equal areas during equal time intervals.
3. The square of the orbital period T is proportional to the cube of the average orbital radius r .

With the help of a labelled diagram, illustrate Kepler's second law for the planets in our Solar System.



[2]

18. Hubble's law can be used to estimate the age of the universe. Fig. 23 shows some of Hubble's early measurements of nearby galaxies plotted on a v against d graph, where v is the recessional speed of a galaxy and d is its distance from us. Measurements of distant galaxies taken over the last 85 years have refined the value of H_0 to be $68 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

- i. Suggest why measurements for our nearest galaxies can deviate from the current Hubble's law trend line.

[1]

- ii. Suggest why measurements for galaxies at the largest distances deviate from the Hubble's law trend line.

[1]

5.4 Gravitational Fields

19. The diagram below shows the Earth in space.



- i. On the diagram above, draw a minimum of **four** gravitational field lines to map out the gravitational field pattern around the Earth.

[1]

- ii. On the same diagram above, show **two** different points where the gravitational potential is the same. Label these points **X** and **Y**.

[1]

20. Fig. 3.2 shows a binary star where the masses of the stars are $4m$ and m .

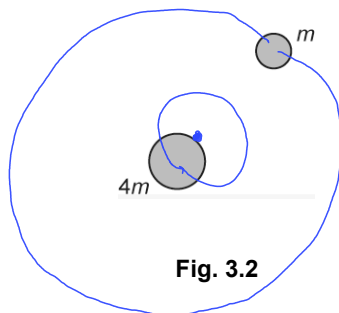
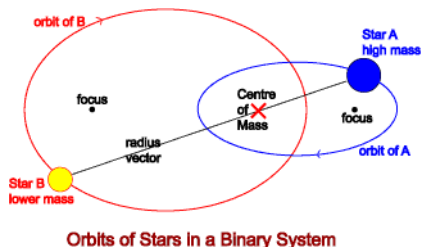


Fig. 3.2

- i. The centre of mass of the binary star lies at the surface of the star of mass $4m$. Draw on **Fig. 3.2** two circles to represent the orbits of **both** stars.

[1]

- ii. Explain why the smaller mass star travels faster in its orbit than the larger mass star.

[2]

5.4 Gravitational Fields

21. Describe the similarities and the differences between the gravitational field of a point mass and the electric field of a point charge.

.....

.....

.....

.....

.....

[3]

22. Kepler's third law can be applied to a satellite in a geostationary orbit around the Earth.

- i. Complete the equation for Kepler's third law below.
You do not need to define any of the terms.

$$\text{.....} = \frac{4\pi^2}{GM} \text{.....} \quad [1]$$

- ii. The mass of Earth is 6.0×10^{24} kg.
Calculate the radius of the circular path of a satellite in a geostationary orbit around the Earth.

radius = m **[2]**

23. Determine the average density of the Earth. The radius of the Earth is 6400 km.

average density = kg m^{-3} **[3]**

5.4 Gravitational Fields

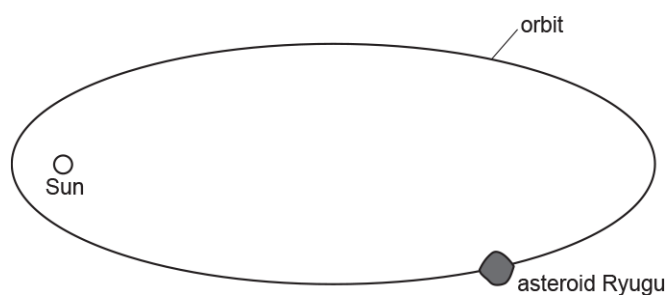
24. Explain what is meant by the statement:

The gravitational potential at the Earth's surface is $-62.7 \times 10^6 \text{ J kg}^{-1}$.

[3]

25 (a). In June 2018, the spacecraft Hayabusa2 arrived at an asteroid called Ryugu.

The asteroid orbits the Sun in an elliptical orbit as shown below.



The diagram is **not** drawn to scale.

(i) Indicate with a letter **X** on the orbit where the asteroid would be moving at maximum speed.

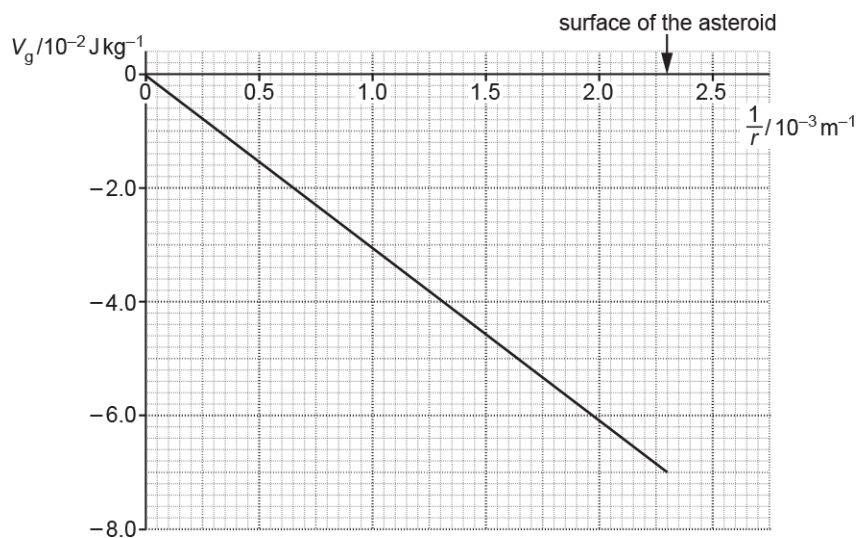
[1]

1. Use Kepler's **second law** to explain your answer to (a)(i) .

[2]

5.4 Gravitational Fields

(b). The gravitational potential at a distance r from the centre of the asteroid Ryugu is V_g . The graph of V_g against $\frac{1}{r}$ for the asteroid is shown below.



i. Define **gravitational potential**.

[1]

ii. Show that the magnitude of the gradient of the graph is equal to GM , where M is the mass of the asteroid and G is the gravitational constant.

[1]

iii. Use the gradient of the graph to show that the mass M of the asteroid is about 4.6×10^{11} kg.

$M = \dots\dots\dots$ kg [2]

(c). In October 2018, the probe Mobile Asteroid Surface Scout (MASCOT) was released from **rest** from the Hayabusa2 spacecraft from a distance of 600 m from the centre of the asteroid.

Assume that the spacecraft was stationary relative to the asteroid when MASCOT was dropped.

Use information from (b) to calculate the speed of the impact v when MASCOT landed on the surface of the asteroid.

$v = \dots\dots\dots$ m s⁻¹ [3]

26. Observations of the planets led to Kepler's three empirical laws:

- 1. The orbit of a planet is an ellipse with the Sun at one focus.
- 2. A line joining a planet and the Sun sweeps out equal areas during equal time intervals.
- 3. The square of the orbital period T is proportional to the cube of the average orbital radius r .

Three exoplanets orbit the star KIC 11442793. Measurements of average orbital radius r and period T for the exoplanets are shown in the table.

r / AU	T / days
0.0881	8.719
0.520	124.9
0.996	331.6

It is suggested that the relationship between T and r is given by Kepler's third law:
 $T^2 \propto r^3$

Propose and carry out a test to check if the relationship is true for the three exoplanets.

Test proposed:

Working:

Conclusion:

5.4 Gravitational Fields

27. This question is about a simple pendulum made from a length of string attached to a mass (bob). For oscillations of small amplitude, the acceleration a of the pendulum bob is related to its displacement x by the expression

$$a = -\left(\frac{g}{L}\right)x$$

where g is the acceleration of free fall and L is the length of the pendulum.
The pendulum bob oscillates with simple harmonic motion.

A student conducts an experiment in the laboratory to investigate the small amplitude oscillations of a pendulum of a mechanical clock. Each 'tick' of the clock corresponds to **half** a period.

- i. Show that the length of the pendulum required for a tick of 1.0 s is about 1 m.

[2]

- ii. If the pendulum clock were to be used on the Moon, explain whether this clock would run on time compared with an identical clock on the Earth.

[2]

28. The galaxies in the Universe may be assumed to be distributed uniformly through space.

In this model, the separation between two neighbouring galaxies is 1.4×10^{23} m and each galaxy occupies a cube of space of volume 2.7×10^{69} m³ as shown in Fig. 24.2.

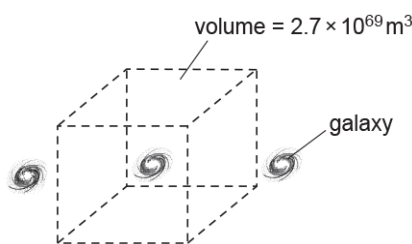


Fig. 24.2

There are on average 10^{11} stars in each galaxy and the mass of an average star is about 2.0×10^{30} kg.

- i. Estimate the gravitational force between two neighbouring galaxies.

force = N [2]

5.4 Gravitational Fields

- ii. Show that the mean density of the Universe is about $7 \times 10^{-29} \text{ kg m}^{-3}$.

[1]

- iii. Suggest why the actual mean density of the Universe is different from the value calculated in (ii).

[1]

29. Algol is a triple-star system, with stars Aa1, Aa2 and Aa3 orbiting each other. This triple-star is 90 light-years from the Earth.

Here is some data on the star Aa1.

- radius = $(1.90 \pm 0.14) \times 10^9 \text{ m}$
- mass = $(6.31 \pm 0.42) \times 10^{30} \text{ kg}$.

Calculate the gravitational field strength g at the surface of Aa1 to 3 significant figures. Include the absolute uncertainty in your answer. Assume that the other stars of the system exert negligible gravitational force on Aa1.

$g = \dots\dots\dots \pm \dots\dots\dots \text{ N kg}^{-1}$ [4]

5.4 Gravitational Fields

30 (a). The apparatus shown in **Fig. 20.1** is used to investigate the variation of the product PV with temperature in the range $20\text{ }^{\circ}\text{C}$ to $100\text{ }^{\circ}\text{C}$. The pressure exerted by the air is P and the volume of air inside the flask is V .

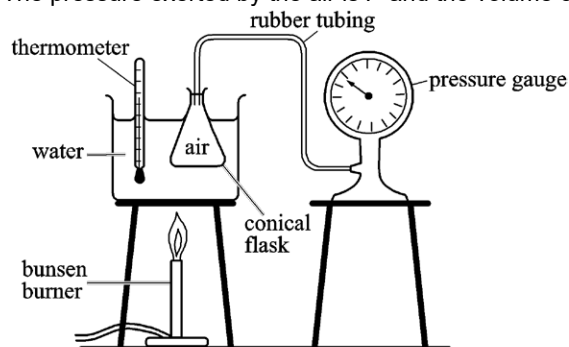


Fig. 20.1

Describe how this apparatus can be set up and used to ensure accurate results.

[4]

(b). An investigation similar to that shown in **Fig. 20.1** gives measurements of the pressure P , volume V and temperature θ in degrees Celsius of a fixed mass of gas.

The results are used to plot the graph of PV against θ shown in **Fig. 20.2**.

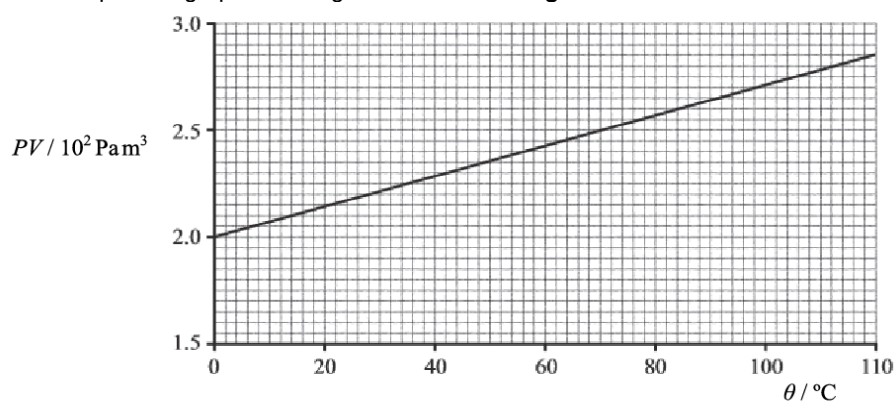


Fig. 20.2

- i. Explain, in terms of the motion of particles, why the graph does **not** go through the origin.

[2]

5.4 Gravitational Fields

- ii. The mass of a gas particle is 4.7×10^{-26} kg. Use the graph in **Fig 20.2** to calculate

1. the mass of the gas

mass = kg

2. the internal energy of the gas at a temperature of 100°C .

internal energy = J **[4]**

31. The International Space Station (ISS) circles the Earth at a height of 4.0×10^5 m.

Its mass is 4.2×10^5 kg.

The radius of the Earth is 6.4×10^6 m.

- i. Show that the speed of the ISS in orbit is about 8 km s^{-1} .

[3]

- ii. Calculate the total energy of the ISS.

total energy = J **[2]**

32. A satellite is in a circular geostationary orbit around the centre of the Earth. The satellite has both kinetic energy and gravitational potential energy.

The mass of the satellite is 2500 kg and the radius of its circular orbit is 4.22×10^7 m.

The mass of the Earth is 5.97×10^{24} kg.

- Describe some of the features of a geostationary orbit.
- Calculate the **total** energy of the satellite in its geostationary orbit.

5.4 Gravitational Fields

33. This question is about a space probe which is in orbit around the Sun.

The space probe has mass 810 kg. The orbital radius of the space probe is 1.5×10^{11} m. The orbital period of the space probe around the Sun is 3.16×10^7 s. The mass of the Sun is 2.0×10^{30} kg.

- i. Show that the magnitude of the gravitational potential energy of the space probe is about 7×10^{11} J.

[2]

- ii. Show that the kinetic energy of the space probe is half the value of your answer to (i).

[3]

- iii. Calculate the total energy of the space probe.

total energy = J[1]

34 (a). An isotope of polonium-213 ($^{213}_{84}\text{Po}$) first decays into an isotope of lead-209 ($^{209}_{82}\text{Pb}$) and this lead isotope then decays into the stable isotope of bismuth (Bi).

Fig. 24 shows two arrows on a neutron number N against proton number Z chart to illustrate these two decays.

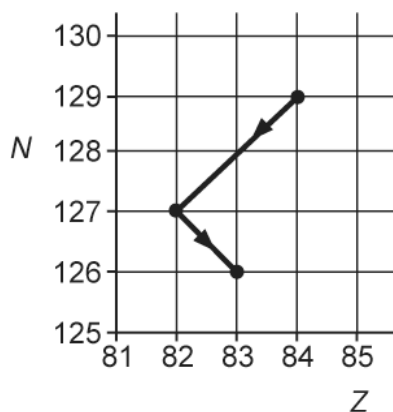


Fig. 24

5.4 Gravitational Fields

Complete the nuclear decay equations for

- i. the polonium isotope



- ii. the lead isotope.



(b). A pure sample of polonium-213 is being produced in a research laboratory.

The half-life of ${}_{84}^{213}\text{Po}$ is very small compared with the half-life of ${}_{82}^{209}\text{Pb}$.

After a very short time, the ionising radiation detected from the sample is mainly from the beta-minus decay of the lead-209 nuclei.

- i. Briefly describe and explain an experiment that can be carried out to confirm the beta-minus radiation emitted from the lead nuclei.

[2]

5.4 Gravitational Fields

- ii. The activity of the sample of $^{209}_{82}\text{Pb}$ after 7.0 hours is 12 kBq.

The half-life of $^{209}_{82}\text{Pb}$ is 3.3 hours.

Calculate the initial number of lead-209 nuclei in this sample.

number of nuclei = [4]

35. A satellite moves in a circular orbit of radius 15 300 km from the centre of the Earth.

- i. State one of the main benefits satellites have on our lives.

[1]

- ii. Calculate the gravitational field strength g at the radius of 15 300 km.

$g = \dots\dots\dots \text{N kg}^{-1}$ [2]

- iii. Calculate the period of the orbiting satellite.

period = $\dots\dots\dots \text{s}$ [3]

5.4 Gravitational Fields

36. * A supply rocket, with its engines shut down, is trying to dock with the International Space Station. Initially it is moving in the same circular orbit above the Earth and at the same speed as the ISS. The two craft are separated by a distance of a few kilometres. The rocket is behind the ISS. It can move closer to the ISS using the following procedure.

The rocket engines are fired in reverse for a few seconds to slow the rocket down. This action causes the rocket to fall into an orbit nearer to the Earth.

After an appropriate time, the rocket engines are fired forwards for a few seconds to move the rocket back into the original orbit closer to the ISS.

Use your knowledge of gravitational forces and uniform motion in a circular orbit to explain the physics of this procedure.

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[6]

37. The table below shows some data for Mercury and Pluto.

	Mass / kg	Radius / m	Mean distance from Sun / m
Mercury	3.30×10^{23}	2.44×10^6	57.9×10^9
Pluto	0.131×10^{23}	1.19×10^6	5910×10^9

- i. Show that the escape velocity v of a gas molecule on the surface of Pluto is given by the equation

$$v = \sqrt{\frac{2GM}{r}}$$

where M is the mass of Pluto and r is its radius.

[2]

- i. Calculate the escape velocity v of gas molecules on the surface of Pluto.

$v = \dots\dots\dots \text{ m s}^{-1}$ [1]

- ii. Explain why Mercury has no atmosphere whilst Pluto still has a thin atmosphere. Use data from the table to support your explanation.

[3]

5.4 Gravitational Fields

38. Phobos is one of the two moons orbiting Mars. Fig. 17.1 shows Phobos and Mars.

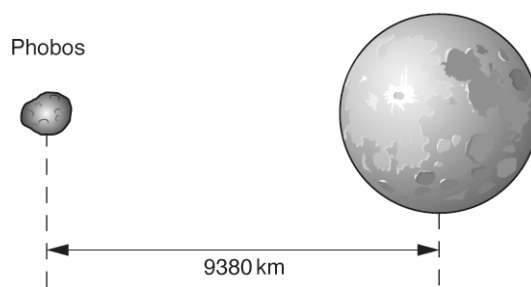


Fig. 17.1

The orbit of Phobos may be assumed to be a circle. The centre of Phobos is at a distance 9380 km from the centre of Mars and it has an orbital speed $2.14 \times 10^3 \text{ m s}^{-1}$.

i.

On Fig. 17.1, draw an arrow to show the direction of the force which keeps Phobos in its orbit.

[1]

ii. Calculate the orbital period T of Phobos.

$T =$

s [2]

iii. Calculate the mass M of Mars.

$M =$

kg [3]

39. A binary star is a pair of stars which move in circular orbits around their common centre of mass.

In this question consider the stars to be point masses situated at their centres.

Fig. 3.1 shows a binary star where the mass of each star is m . The stars move in the same circular orbit.

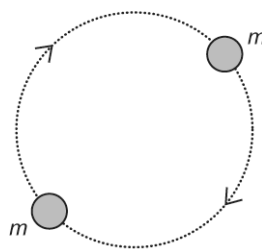


Fig. 3.1

5.4 Gravitational Fields

- i. Explain why the stars of equal mass must always be diametrically opposite as they travel in the circular orbit.

[2]

- ii. The centres of the two stars are separated by a distance of $2R$ equal to 3.6×10^{10} m, where R is the radius of the orbit. The stars have an orbital period T of 20.5 days. The mass of each star is given by the equation

$$m = \frac{16\pi^2 R^3}{GT^2}$$

where G is the gravitational constant.

Calculate the mass m of each star in terms of the mass $M_{\odot}$ of the Sun.

$$1 \text{ day} = 86400 \text{ s}$$

$$M_{\odot} = 2.0 \times 10^{30} \text{ kg}$$

$$m = \quad \quad \quad M_{\odot} \text{ [3]}$$

- iii. The stars are viewed from Earth in the plane of rotation. The stars are observed using light that has wavelength of 656 nm in the laboratory. The observed light from the stars is Doppler shifted.

Calculate the maximum change in the observed wavelength $\Delta\lambda$ of this light from the orbiting stars. Give your answer in nm.

$$\Delta\lambda = \quad \quad \quad \text{nm [2]}$$

5.4 Gravitational Fields

40. This question is about helium in the atmosphere of the Earth.

Experiment shows that most of the Earth's atmosphere is contained within a very thin shell around the surface of the Earth. Less than 0.0001% of this is helium.

The height of the atmosphere is negligible compared with the radius R of the Earth.

- i. Show that the minimum speed v_E required for an atom or molecule to escape from the top of the Earth's atmosphere is given by the expression

$$v_E = \sqrt{2gR}.$$

[3]

- ii. The radius R of the Earth is 6.4×10^6 m. Calculate this escape speed v_E .

$$v_E = \dots\dots\dots \text{ m s}^{-1} \quad \text{[1]}$$

- iii. Calculate the temperature T in kelvin required at the top of the Earth's atmosphere for the root mean square speed $c_{\text{r.m.s.}}$ of the helium atoms there to equal this escape speed.

Molar mass of helium = $0.004 \text{ kg mol}^{-1}$

$$T = \dots\dots\dots \text{ K} \quad \text{[3]}$$

- iv. Fig. 1 shows the distribution of the speeds of the atoms of an ideal gas.

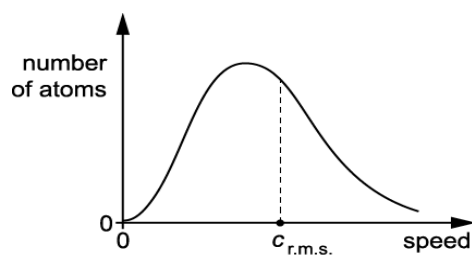


Fig. 1

Use your knowledge of the kinetic theory of gases to describe the shape of this distribution and explain why some helium is able escape from the Earth.

[4]

- v. Over a very long period of time all of the helium should have escaped from the Earth. Suggest why there is still a small amount of helium, about 0.0001%, in the Earth's atmosphere.

[2]

END OF QUESTION PAPER